Midterm Exam Calculus 1

22 september 2009, 9.00-11.00.

Write on each page your name and student number, and on the first page your seminar group. The use of annotations, books and calculators is not permitted in this examination. Excercise 2 yields at most one point; the maximum score for the other exercises is 2 points. Total: 9 + 1 (free) = 10. All answers must be supported by arguments/work. Success.

1. Prove that if $n \geq 1$ is a positive integer, then

$$1^3 + 2^3 + ... + n^3 = n^2(n+1)^2/4$$

2. Sketch (in the complex plane) the set of points z given by

$$|z+1| = |z-i|$$

3. Determine all complex numbers z satisfying

$$z^3 = -2 + 2i$$

4. Evaluate the limit, if it exists.

$$\lim_{h \to 0} \frac{\sqrt{(1+h)} - 1}{h}$$

5. We consider the function

$$f(x) = x \sin \frac{1}{x}$$

where $x \neq 0$

   (a) Give the $\epsilon - \delta$ definition of a limit.

   (b) Show, using this definition, that

$$\lim_{x \to 0} f(x) = 0$$